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# Identification of amplitude-dependent flutter derivatives of bridge deck via free coupled vibration test of sectional model

Le-Dong Zhu [1,2,3*], Cheng Qian [2], Quan-Shun Ding [1,2,3]

*[1]State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai, China, Ledong@tongji.edu.cn*
*[2]Department of Bridge Engineering, Tongji University, Shanghai, China*
*[3]Key Laboratory of Transport Industry of Bridge Wind Resistance Technology, Tongji University, Shanghai, China*

**SUMMARY:**
Scanlan's linear self-excited force model has been widely used in linear flutter research of bridges, and can be expanded to nonlinear flutter by regarding its flutter derivatives as functions of not only the reduced frequency but also the vibration amplitude. A DOF-equilibrium equation-based method was established for identifying amplitude-dependent flutter derivatives of nonlinear self-excited forces via free coupled vibration tests of 2-DOF sectional model. The proposed identification method can also consider the nonlinearity of the mechanical damping and stiffness of the sectional model system. The amplitude-dependent behaviours of flutter derivatives were then discussed by taking a centrally-slotted box deck as an example. The results show that among the four torsional motion related flutter derivatives, $H_2^*$ and $A_2^*$ exhibit strong amplitude-dependent characters, whilst the amplitude dependence of $H_3^*$ is very weak and $A_3^*$ depends on the amplitude to some extent. Moreover, the amplitude dependences of all the four vertical motion-related flutter derivatives, $H_1^*$, $H_4^*$, $A_1^*$ and $A_4^*$ are quite weak. The feasibility and reliability of this method were verified by comparing the calculated and measured nonlinear flutter responses of sectional modal system.

*Keywords: bridge deck, amplitude-dependent flutter derivative, free coupled vibration test of sectional model*

## 1. INTRODUCTION

Free vibration tests of sectional model are often used for flutter derivative identification. However, most of current identification method based on the free vibration tests are under the assumption of linear flutter (Ding et al., 2010) or for nonlinear single DOF (degree of freedom) torsional flutter(Zhu and Gao, 2016). For flutter derivative identification of nonlinear coupled flutter, some researchers introduced approximate interrelationships among different flutter derivatives of coupled flutter to reduce the number of unknown variable number (Wang et al., 2020) , or carried out additional pure torsional vibration test to obtain amplitude-dependent torsional motion-related flutter derivatives $A_2^*$ and $A_3^*$, then identified other ones through free coupled vibration test (Wu et al., 2020). The shortcomings of the above two types of identification methods are obvious. On this account, identification method for amplitude-dependent flutter derivatives of nonlinear self-excited forces only based on free coupled vibration tests of 2-DOF sectional model was investigated in this study.

## 2. FUNDAMENTAL PRINCIPLE OF THE PROPOSED IDENTIFICATION METHOD

A 2-DOF sectional model system under a certain wind speed is generally a weak nonlinear system, and its nonlinearity is mainly reflected in damping. Its nonlinear equations of self-excited vibration can be expressed as follows in the light of equivalent linearization theory.

$$\ddot{h} + 2\omega_{v0}(a'_h)\xi_{v0}(a'_h)\dot{h} + \omega_{v0}^2(a'_h)h = \omega H_1(K,a'_h)\dot{h} + \omega H_2(K,a_\alpha)\dot{\alpha} + \omega^2 H_3(K,a_\alpha)\alpha + \omega^2 H_4(K,a'_h)h \tag{1a}$$

$$\ddot{\alpha} + 2\omega_{t0}(a_\alpha)\xi_{t0}(a_\alpha)\dot{\alpha} + \omega_{t0}^2(a_\alpha)\alpha = \omega A_1(K,a'_h)\dot{h} + \omega A_2(K,a_\alpha)\dot{\alpha} + \omega^2 A_3(K,a_\alpha)\alpha + \omega^2 A_4(K,a'_h)h \tag{1b}$$

Where, $\omega$ is the circular frequency of vibration at the wind speed of *U,* which is actually a damped circular frequency; $\omega_{v0}$ , $\omega_{t0}$ and $\xi_{v0}$ , $\xi_{t0}$ are the undamped circular frequencies and damping ratios of vertical and torsional modes at zero wind speed, they are also amplitude dependent in general (Gao and Zhu, 2015); $H_1 = \rho B^2 H_1^*/m$ ; $H_2 = \rho B^3 H_2^*/m$ ; $H_3 = \rho B^3 H_3^*/m$ ; $H_4 = \rho B^2 H_4^*/m$ ; $A_1 = \rho B^3 A_1^*/I_m$ ; $A_2 = \rho B^4 A_2^*/I_m$ ; $A_3 = \rho B^4 A_3^*/I_m$ ; $A_4 = \rho B^3 A_4^*/I_m$ . $H_i^*$ and $A_i^*$ are the amplitude-dependent flutter derivatives and are functions of the reduced frequency ( $K = B\omega/U$ ) and vibration amplitude $a'_h = a_h/D$ or $a_\alpha$ ; *D* is the deck depth and *B* is the deck width. *h* and $\alpha$ are, respectively, the dynamic vertical and torsional displacements, and can be expressed as follows:

$$h = h_v + h_t = A_{0hv}e^{-\xi_v\omega_v t}\cos(\omega_v t + \beta_{hv}) + A_{0ht}e^{-\xi_t\omega_t t}\cos(\omega_t t + \beta_{ht}) = a_{hv}(t)\cos\theta_v(t) + \phi_t a_{\alpha t}(t)\cos\left(\theta_t(t) + \Delta\theta_t\right) \tag{2a}$$

$$\alpha = \alpha_v + \alpha_t = A_{0\alpha v}e^{-\xi_v\omega v_v t}\cos(\omega_v t + \beta_{\alpha v}) + A_{0\alpha t}e^{-\xi_t\omega_t t}\cos(\omega_t t + \beta_{\alpha t}) = \phi_v a_{hv}(t)\cos\left(\theta_v(t) + \Delta\theta_v\right) + a_{\alpha t}(t)\cos\theta_t(t) \tag{2b}$$

Where, $h_i$, $\alpha_i$, $\xi_i$, $\omega_i$, $\beta_{hi}$, $\beta_{\alpha i}$, $A_{0hi}$, $A_{0\alpha i}$, $a_{hi}$, $a_{\alpha i}$ (*i*=*v*, *t*) are, respectively, the vertical and torsional displacements contributed by the *i*th mode, the damping ratio and circular frequency of the *i*th mode, the initial phases and amplitudes, and the time-varying amplitude of $h_i$ and $\alpha_i$, where, *i*=*v* represents the first mode dominated by the vertical vibration, and *i*=*t* represents the second mode dominated by the torsional vibration; $\theta_i = \omega_i t$ , $\dot{\theta}_i = \omega_i$ , $\Delta\theta_i$ and $\phi_i$ are the phase difference and the mode shape coefficient between the response components in two DOFs, where, $\Delta\theta_v=\beta_{\alpha v}-\beta_{hv}$, $\Delta\theta_t=\beta_{ht}-\beta_{\alpha t}$, $\phi_v= a_{\alpha v}/a_{hv}\approx A_{0\alpha v}/U_{0hv}$, $\phi_t=a_{ht}/a_{\alpha t}\approx U_{0ht}/U_{0\alpha t}$.

Previous research results show that $\Delta\theta_i$ and $\phi_i$ are approximately independent of vibration amplitude, and can then be obtained at first by directly fitting the measured time histories of $\hat{h}(t)$ and $\hat{\alpha}(t)$ under the temporary approximate assumption of constant frequencies and damping ratios. Then, the modal signals of $h_i(t)$ , $\alpha_i(t)$ , (*i*=*v*, *t*) can be extracted from $\hat{h}(t)$ and $\hat{\alpha}(t)$ by using a decomposition method based on Hilbert transform, and the amplitude-dependent modal frequencies and damping ratios can then be identified out with the method introduced in Gao and Zhu (2015). Noted that Eqs.(1) are also applicable to any one of the two extracted single-mode responses, one can thus get the two DOF-equilibrium equations for the vertical-dominated mode at $K = B\omega_v/U_v$ , and another two for the torsional-dominated mode at the same $K = B\omega_t/U_t$ . Normally, $\omega_t > \omega_v$ , therefore, $U_t > U_v$ . For example, Eq. (3) is the equilibrium equation in the vertical DOF for the vertical-dominated mode. Then, one can obtain the relationships between flutter derivatives and system parameters, as shown as Eqs.(4) and Eqs.(5)

$$\begin{aligned}\ddot{h}_v + 2\omega_{v0}(a'_{hv})\xi_{v0}(a'_{hv})\dot{h}_v + \omega_{v0}^2(a'_{hv})h_v = \omega_v(t)\left[H_1 + H_2\cdot\phi_v\left(\cos(\Delta\theta_v) - \xi_v(t)\sin(\Delta\theta_v)\right) + H_3\cdot\phi_v\sin(\Delta\theta_v)\right]\dot{h}_v + \\ \omega_v^2(t)\left[-H_2\phi_v\sin(\Delta\theta_v)\left(1+\xi_v^2(t)\right) + H_3\cdot\phi_v(\cos(\Delta\theta_v) + \xi_v(t)\sin(\Delta\theta_v)) + H_4\right]h_v\end{aligned} \tag{3}$$

$$\begin{bmatrix} 1 & \phi_v\left(\cos(\Delta\theta_v)-\xi_v(a'_{hv})\sin(\Delta\theta_v)\right) & \phi_v\sin(\Delta\theta_v) & 0 \\ 0 & -\phi_v\sin(\Delta\theta_v)\left(1+\xi_v^{\,2}(a'_{hv})\right) & \phi_v\left(\cos(\Delta\theta_v)+\xi_v(a'_{hv})\sin(\Delta\theta_v)\right) & 1 \end{bmatrix}\begin{pmatrix} H_1(K,a'_{hv}) \\ H_2(K,a_{\alpha v}) \\ H_3(K,a_{\alpha v}) \\ H_4(K,a'_{hv}) \end{pmatrix}=\begin{pmatrix} \dfrac{2\omega_{v0}(a'_{hv})\xi_{v0}(a'_{hv})}{\omega_v(a'_{hv})}-\dfrac{2\xi_v(a'_{hv})}{\sqrt{1-\xi_v^2(a'_{hv})}} \\ \dfrac{\omega_{v0}^2(a'_{hv})}{\omega_v^{\,2}(a'_{hv})}-\dfrac{1}{1-\xi_v^{\,2}(a'_{hv})} \end{pmatrix} \quad (4a)$$

$$\begin{bmatrix} \dfrac{1}{\phi_v}\left(\cos(\Delta\theta_v)+\xi_v(a'_{hv})\sin(\Delta\theta_v)\right) & 1 & 0 & -\dfrac{1}{\phi_v}\sin(\Delta\theta_v) \\ \dfrac{1}{\phi_v}\sin(\Delta\theta_v)\left(1+\xi_v^{\,2}(a'_{hv})\right) & 0 & 1 & \dfrac{1}{\phi_v}\left(\cos(\Delta\theta_v)-\xi_v(a'_{hv})\sin(\Delta\theta_v)\right) \end{bmatrix}\begin{pmatrix} A_1(K,a'_{hv}) \\ A_2(K,a_{\alpha v}) \\ A_3(K,a_{\alpha v}) \\ A_4(K,a'_{hv}) \end{pmatrix}=\begin{pmatrix} \dfrac{2\omega_{t0}(a_{\alpha v})\xi_{\alpha}(a_{\alpha v})}{\omega_v(a'_{hv})}-\dfrac{2\xi_v(a'_{hv})}{\sqrt{1-\xi_v^{\,2}(a'_{hv})}} \\ \dfrac{\omega_{t0}^2(a_{\alpha v})}{\omega_v^{\,2}(a'_{hv})}-\dfrac{1}{1-\xi_v^{\,2}(a'_{hv})} \end{pmatrix} \quad (4b)$$

$$\begin{bmatrix} 1 & \dfrac{1}{\phi_t}\left(\cos(\Delta\theta_t)+\xi_t(a_{\alpha t})\sin(\Delta\theta_t)\right) & -\dfrac{1}{\phi_t}\sin(\Delta\theta_t) & 0 \\ 0 & \dfrac{1}{\phi_t}\sin(\Delta\theta_t)\left(1+\xi_t^{\,2}(a_{\alpha t})\right) & \dfrac{1}{\phi_t}\left(\cos(\Delta\theta_t)-\xi_t(a_{\alpha t})\sin(\Delta\theta_t)\right) & 1 \end{bmatrix}\begin{pmatrix} H_1(K,a'_{ht}) \\ H_2(K,a_{\alpha t}) \\ H_3(K,a_{\alpha t}) \\ H_4(K,a'_{ht}) \end{pmatrix}=\begin{pmatrix} \dfrac{2\omega_{v0}(a'_{ht})\xi_{v0}(a'_{ht})}{\omega_t(a_{\alpha t})}-\dfrac{2\xi_t(a_{\alpha t})}{\sqrt{1-\xi_t^{\,2}(a_{\alpha t})}} \\ \dfrac{\omega_{v0}^2(a'_{ht})}{\omega_t^{\,2}(a_{\alpha t})}-\dfrac{1}{1-\xi_t^{\,2}(a_{\alpha t})} \end{pmatrix} \quad (5a)$$

$$\begin{bmatrix} \phi_t\left(\cos(\Delta\theta_t)+\xi_t(a_{\alpha t})\sin(\Delta\theta_t)\right) & 1 & 0 & \phi_t\sin(\Delta\theta_t) \\ -\phi_t\sin(\Delta\theta)\left(1+\xi_t^{\,2}(a_{\alpha t})\right) & 0 & 1 & \phi_t\left(\cos(\Delta\theta_t)+\xi_t(a_{\alpha t})\sin(\Delta\theta_t)\right) \end{bmatrix}\begin{pmatrix} A_1(K,a'_{ht}) \\ A_2(K,a_{\alpha t}) \\ A_3(K,a_{\alpha t}) \\ A_4(K,a'_{ht}) \end{pmatrix}=\begin{pmatrix} \dfrac{2\omega_{t0}(a_{\alpha t})\xi_{t0}(a_{\alpha t})}{\omega_t(a_{\alpha t})}-\dfrac{2\xi_t(a_{\alpha t})}{1-\xi_t^{\,2}(a_{\alpha t})} \\ \dfrac{\omega_{t0}^2(a_{\alpha t})}{\omega_t^2(a_{\alpha t})}-\dfrac{1}{1-\xi_t^{\,2}(a_{\alpha t})} \end{pmatrix} \quad (5b)$$

The solution of the above equations should be carried out according to the following 3 steps. (1) Solving the 8 linear flutter derivatives, $H_i(K)$ ($i$=1,…,4) and $A_i(K)$ ($i$=1,…,4), at first according to the above 8 equations by using vibration signals with small amplitudes. (2) As it is well known that the vibration amplitude in coupled DOF of the vertical-dominated mode, i.e., $a_{\alpha v}$, is generally quite small, therefore, the amplitude-dependent $H_1(K,a'_{hv})$, $H_4(K,a'_{hv})$ and $A_1(K,a'_{hv})$, $A_4(K,a'_{hv})$ at different levels of $a'_{hv}$ can be solved according to Eqs.(4a) and Eqs.(4b) by approximately replacing $H_2(K,a_{\alpha v})$, $H_3(K,a_{\alpha v})$ and $A_2(K,a_{\alpha v})$, $A_3(K,a_{\alpha v})$ with the corresponding linear ones of $H_2(K)$, $H_3(K)$, $A_2(K)$ and $A_3(K)$ which have been obtained already in the first step. (3) Previous research results reported by some researchers as well as obtained by the authors in this study show that the nonlinearities or amplitude-dependences of $H_1$, $H_4$ and $A_1$, $A_4$ are quite weak, therefore, $H_1(K,a'_{ht})$, $H_4(K,a'_{ht})$ and $A_1(K,a'_{ht})$, $A_4(K,a'_{ht})$ in Eqs.(5a) and Eqs.(5b) can also be approximately replaced with the corresponding linear ones of $H_1(K)$, $H_4(K)$, $A_1(K)$ and $A_4(K)$ which are already known. Thus, the amplitude-dependent $H_2(K,a_{\alpha t})$, $H_3(K,a_{\alpha t})$ and $A_2(K,a_{\alpha t})$, $A_3(K,a_{\alpha t})$ at different levels of $a_{\alpha t}$ can be solved according to Eqs.(5a) and Eqs.(5b). It should be noted that interpolations or sometimes extrapolations of $\Delta\theta_i$, $\phi_i$, $\xi_{i0}$, $\omega_{i0}$, $\xi_i$, $\omega_i$ $(i=v,t)$ are generally needed during the identification process because the $U_v$ and $U_t$, which meet the requirement of $B\omega_v/U_v = B\omega_t/U_t = K$ is often not coincide with the testing wind speeds.

## 3. APPLICATION EXAMPLE

The foregoing-introduced identification method for amplitude-dependent flutter derivatives of nonlinear coupled self-excited forces was applied to a centrally-slotted bridge deck as shown in Fig.1. The results show that the amplitude dependence of $H_2^*$ and $A_2^*$ is strong, and those of $H_3^*$ is very weak. $A_3^*$ depends on the amplitude to a certain extent. The amplitude dependences of $H_1^*$, $H_4^*$, $A_1^*$ and $A_4^*$ are quite weak. The results of the major flutter derivatives at a wind effective attack angle of -5° are shown in Fig.2. Furthermore, the nonlinear flutter displacement responses of 2-DOF sectional model calculated with the identified flutter derivatives agree well with the measured ones.

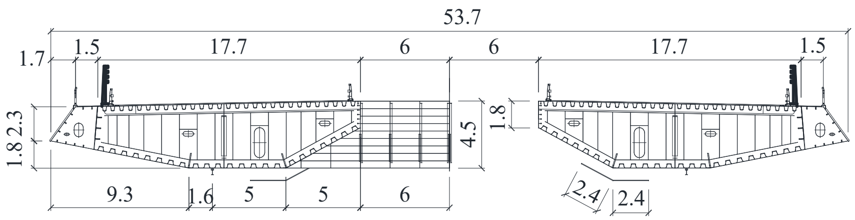


**Figure 1.** Cross section of the example centrally-slotted bridge deck

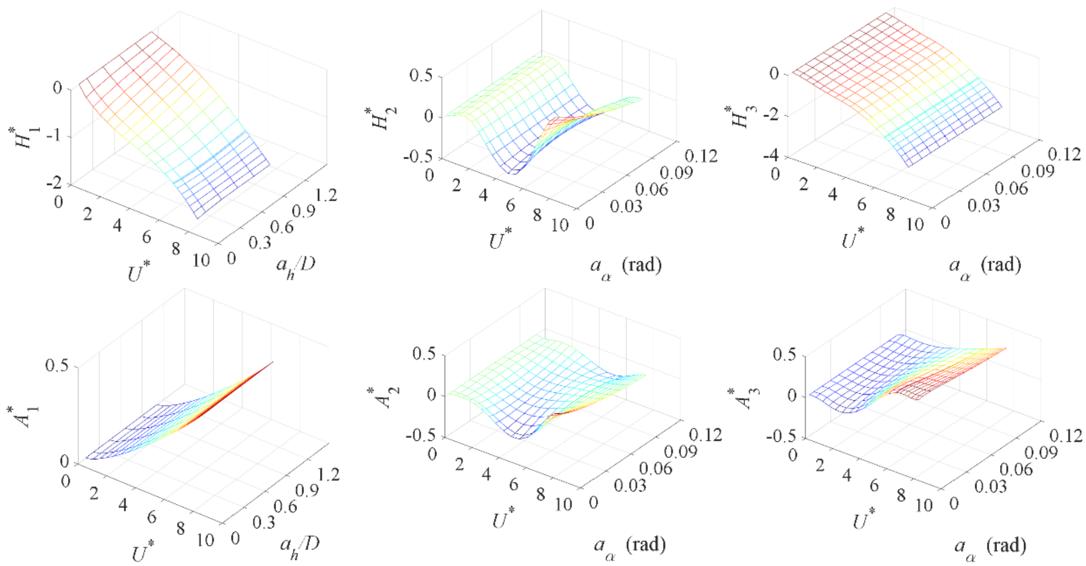


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**Figure 2.** Identified results of the major flutter derivatives ($\alpha_e = -5°$)

## 4. CONCLUSIONS

A DOF-equilibrium equation-based identification method for amplitude-dependent flutter derivatives based on 2-DOF free vibration wind tunnel test of sectional model was presented and applied to practice. The feasibility and reliability of this method were verified by comparing the calculated and measured nonlinear flutter responses of sectional modal system.

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**REFERENCES**

Ding Q., Zhou Z., Zhu L., and Xiang, H., 2010. Identification of flutter derivatives of bridge decks with free vibration technique. Journal of Wind Engineering and Industrial Aerodynamics 98(12): 911-918.

Gao, G.Z. and Zhu, L.D., 2015. Nonlinearity of mechanical damping and stiffness of a spring-suspended sectional model system for wind tunnel tests. Journal of Sound and Vibration 355: 369- 391.

Wang, Y., Chen, X., and Li, Y., 2020. Nonlinear self-excited forces and aerodynamic damping associated with vortex-induced vibration and flutter of long span bridges. Journal of Wind Engineering and Industrial Aerodynamics 204: 104207.

Wu, B., Chen, X.Z., Wang, Q., et al., 2020. Characterization of vibration amplitude of nonlinear bridge flutter from section model test to full bridge estimation. Journal of Wind Engineering and Industrial Aerodynamics 197: 104048.